

Asymptotic Safety in Einstein Gravity and Scalar-Fermion Matter

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Abstract. Within the functional renormalization group approach we study the effective QFT of Einstein gravity and one self-interacting scalar coupled to N_f Dirac fermions. We include in our analysis the matter anomalous dimensions induced by all the interactions and analyze the highly non linear beta functions determining the renormalization flow. We find the existence of a non trivial fixed point structure both for the gravity and the matter sector, besides the usual gaussian matter one. This suggest that asymptotic safety could be realized in the gravitational sector *and* in the standard model. Non triviality in the Higgs sector might involve gravitational interactions.

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I. INTRODUCTION

The Einstein theory of gravitational interactions appears to be strongly coupled at Planck scale and is well known to be perturbatively non renormalizable [1]. Therefore, as a quantum theory, is generally considered to be an effective one, nevertheless useful to compute quantum corrections [2] induced by graviton exchanges in low energy processes. At high energies it is generally believed to derive from a fundamental counterpart such as string theory or to be possibly described in terms of spin foams or lattice models. There have been also proposal that gravity may be induced [3] by other degrees of freedom, e.g. matter ones, and recently that gravitational field equations may be derived as thermodynamic properties [4]. Even in this latter scenario where gravity is considered emergent we expect the presence, apart from matter quantum fluctuations, of quantum fluctuations of the gravitational degrees of freedom [5], which may therefore be described in some domain by an effective QFT.

The Higgs sector of the standard model is also regarded as an effective field theory since it cannot be arbitrarily extended to ultraviolet (UV) energies. This is related to the triviality problem which manifests itself even in perturbation theory due to the appearance of a Landau pole when studying the renormalized self-interaction of the Higgs field.

In the context of QFT there is nevertheless the possibility to have an UV complete description of the gravitational and standard model theories, so that they could be considered *fundamental*. The paradigm of *asymptotic safety* has been introduced to provide such a situation, which is realized whenever the renormalization group (RG) analysis of the theory reveals the existence of a fixed point (FP) with a finite number of UV attractive (relevant) directions in the theory space [6].

Analytical tools for the investigations of RG flows of QFT models in the Wilsonian sense [7], leading to functional renormalization group equations [8, 9] are available. In particular increasing evidence that gravity might be asymptotically safe (and therefore a meaningful QFT) comes from the analysis of effective aver-

age action (EAA) truncations which include terms associated to the Einstein-Hilbert and cosmological constant [10], and then scalar matter minimally or non minimally coupled [11, 12], quartic and higher derivative terms of the metric [13–15], free matter of all spin less than two [14, 16, 17].

On the other hand, the possibility that asymptotic safety can solve the triviality as well as the hierarchy problem in the standard model and in particular in the Higgs sector has been recently explored. Several models for interacting scalar-fermion matter have been considered, starting from the most simple ones, both in the symmetric and in the SSB phases [18]. The simplest model does not show the possibility to have non trivial FPs but for models of increasing complexity there exists a range of parameters suitable for asymptotic safety to be realized.

In this letter we report and discuss the results of a study of Einstein gravity interacting with the simplest possible system with one scalar field and N_f Dirac fermions with a $U(N_f)$ symmetric Yukawa interaction, which by itself has been found to be a non asymptotic safe theory. The investigations on such a system started in [19]. For a perturbative approach at one loop with dimensional regularization see also [20].

The goal is twofold. On one hand we would like to see if the inclusion of such an interacting matter can spoil the formation of the non trivial FP structure in the gravity sector. In doing this we also consider for the first time the anomalous dimensions of the matter fields, which have been previously neglected in all asymptotic safety studies which include gravity. On the other hand we want to analyze if in this simplified model the gravitational interaction leads to the possibility to have an asymptotically safe matter sector, eventually removing the triviality problem in the scalar sector.

This might serve as a prelude for a more general analysis involving the Higgs, fermion and gauge fields of the standard model. There is the possibility that the UV completion of such a theory together with gravity might be realized in a satisfactory way in the QFT sense. This would constitute an alternative to considering a scenario where these theories emerge at low energy from superstring

theories or other viable GUT.

II. GRAVITY AND YUKAWA SYSTEM

We adopt Wilson's approach to renormalization and introduce an euclidean EAA with a scale dependence. We truncate it to the form

$$\Gamma_k[\Upsilon] = \int d^4x \sqrt{g}(L_b + L_f + L_g) \quad (1)$$

where the single terms are

$$L_b = \frac{1}{2} Z_\phi \nabla^\mu \phi \nabla_\mu \phi + V(\phi) \quad (2)$$

$$L_f = \frac{i}{2} Z_\psi (\bar{\psi} \gamma^\mu D_\mu \psi - D_\mu \bar{\psi} \gamma^\mu \psi) + i H(\phi) \bar{\psi} \psi \quad (3)$$

$$L_g = -Z \sqrt{g} R [g_{\mu\nu}] \quad (4)$$

The multiplet $\Upsilon^A = (\phi, \psi, \bar{\psi}, g_{\mu\nu})$ contains all the dynamical fields. k is a sliding renormalization scale that parameterizes the RG flow of the EAA. The Newton constant is contained in $Z = 1/16\pi G$, while the cosmological constant is the zero point energy of the potential $V(\phi)$. $H(\phi)$ is a generalized Yukawa interaction. The spinor covariant derivative is $D_\mu \psi = \partial_\mu \psi + 1/2 \omega_{\mu cd} J^{cd} \psi$, where $\omega_{\mu cd}$ is the spin connection and $J^{cd} = 1/4[\gamma^c, \gamma^d]$ are the $O(4)$ generators.

We use the background field method in conjunction to exact renormalization group. The action is expanded around a background Υ using fluctuations $\Sigma^{\hat{A}} = (\varphi, \chi, \bar{\chi}, h_{\mu\nu}, \bar{c}_\mu, c_\mu)$ and define

$$\Gamma_k[\Sigma, \Upsilon] = \Gamma_k[\Upsilon + \Sigma] + L_{g.f.} + L_{gh} \quad (5)$$

We add a gauge fixing term for the diffeomorphisms and a corresponding ghost action

$$L_{g.f.} = \frac{Z}{2\alpha} g^{\mu\nu} F_\mu F_\nu; \quad F_\mu = \left(\delta_\mu^\beta \nabla^\alpha - \frac{1+\beta}{4} g^{\alpha\beta} \nabla_\mu \right) h_{\alpha\beta}$$

$$L_{gh} = \bar{c}_\mu \left(-g^{\mu\nu} \nabla^2 + \frac{\beta-1}{2} \nabla^\mu \nabla^\nu - R_{\mu\nu} \right) c_\nu. \quad (6)$$

Notice that we included the ghosts in the fluctuations multiplet. The $O(4)$ gauge is fixed such that the vierbein is symmetric and all their fluctuations can be written in terms of $h_{\mu\nu}$. There are no $O(4)$ ghosts.

The coarse-graining procedure is achieved using an IR cutoff term

$$\Delta S_k = \int d^4x \sqrt{g} \Sigma^T \mathcal{R}_k[\Upsilon] \Sigma \quad (7)$$

We define the modified propagator $\mathcal{G}_k \equiv (\Gamma_k^{(2,0)}[\Upsilon, 0] + \mathcal{R}_k[\Upsilon])^{-1}$. The apex numbers $(2,0)$ indicate that two functional derivatives w.r.t. the first argument were taken. The flow of the EAA is governed by the exact RG equation (ERGE) [9]

$$k \partial_k \Gamma_k = \frac{1}{2} \text{STr} \mathcal{G}_k \dot{\mathcal{R}}_k = \frac{1}{2} \text{Diagram} \quad (8)$$

where the dot means $t = \text{Log } k/k_0$ derivative and k_0 is a reference scale; $\partial_t = k \partial_k$. We require the cutoff kernel to be a function of the background metric $g_{\mu\nu}$ only, i.e. $\mathcal{R}_k = \mathcal{R}_k[g]$. In particular, the cutoff is required to be a function of the operators $\square = -\nabla^2$ and $i\gamma^\mu D_\mu$, so that any covariant derivative appearing in $\Gamma_k^{(2,0)}[\Upsilon, 0]$ is coarse-grained by $\mathcal{R}_k = \mathcal{R}_k[g]$. The profile function of the cutoff is always the optimized one [21], $R_{k,\text{opt.}}(z) =$

$(k^2 - z)\theta(k^2 - z)$, for $z = \square$, and allows to perform exactly the trace. The choice we use is often referred as ‘‘type-I’’ cutoff in the existing literature [14] and coincides with that of [16]. Other cutoff choices are possible and we will comment more on them in the discussions.

The flow of the functions $V(\phi)$ and $H(\phi)$ is calculated using (8) evaluated in a background for which $\phi = \text{const.}$, $\psi = \text{const.}$ and $g_{\mu\nu} = \delta_{\mu\nu}$. All this, apart for a slightly different choice of the cutoff scheme, goes along the lines of our previous work [19]. In order to calculate the flow of Z_ϕ and Z_ψ , we take two functional derivatives of (8) with respect to Υ^A and Υ^B such that $\Upsilon^A, \Upsilon^B \neq g_{\mu\nu}$. We obtain the flow of the 2-point function

$$k \partial_k \frac{\delta^2 \Gamma_k}{\delta \Upsilon^A \delta \Upsilon^B} = \text{STr} \mathcal{G}_k \frac{\delta \Gamma_k^{(2,0)}}{\delta \Upsilon^A} \mathcal{G}_k \frac{\delta \Gamma_k^{(2,0)}}{\delta \Upsilon^B} \mathcal{G}_k \dot{\mathcal{R}}_k$$

$$- \frac{1}{2} \text{STr} \mathcal{G}_k \frac{\delta^2 \Gamma_k^{(2,0)}}{\delta \Upsilon^A \delta \Upsilon^B} \mathcal{G}_k \dot{\mathcal{R}}_k$$

$$= - \text{Diagram} - \frac{1}{2} \text{Diagram} \quad (9)$$

Matrix indices have been omitted for notational simplicity. The derivatives did not act on \mathcal{R}_k because it is a function of the metric only. The anomalous dimensions are defined as $\eta_\phi = -\dot{Z}_\phi/Z_\phi$ and $\eta_\psi = -\dot{Z}_\psi/Z_\psi$. We use (9) in the background $\phi = \text{const.}$, $\psi = 0$, $g_{\mu\nu} = \delta_{\mu\nu}$ and then move to momentum space. Additionally, we require $V'(\phi) = 0$ to simplify to some extent. Choosing in (9) $\Upsilon^A = \Upsilon^B = \phi$, or $\Upsilon^A = \psi$ and $\Upsilon^B = \bar{\psi}$, we calculate η_ϕ and η_ψ , isolating the coefficients of the operators $Z_\phi \square$ and $i Z_\psi \gamma^\mu D_\mu$, respectively. η_ϕ and η_ψ calculated in this way are *on-shell* and have a ϕ dependence that has to be fixed by physical requirements.

The running \dot{Z} is calculated using heat kernel methods. We set the background to be a 4-sphere with corresponding metric and choose $\phi = \text{const.}$ and $\psi = 0$. (8) is then written as the trace of a function of \square . The Seeley-DeWitt expansion gives the term linear in $R[g_{\mu\nu}]$ of (8), from which we read the beta function \dot{Z} .

III. RG FLOW

Using (9) and the heat kernel technique, we get a system for the quantities $\{\eta_\phi, \eta_\psi, \dot{Z}\}$ which has to be solved. $\bar{Z} = k^{-2} Z$ is the dimensionless Planck mass. η_ϕ, η_ψ and \dot{Z} depend on the constant field configuration ϕ through the functions H and V . We set it to be $\langle \phi \rangle$ in order to agree with the initial truncation (2,3,4) that has constant $Z_{\phi,\psi}$. From (8) we calculate \dot{V} and \dot{H} . It is convenient to use the dimensionless renormalized field $\phi_R = Z_\phi^{1/2} k^{-1} \phi$ and functions $v[\phi_R] = k^{-4} V[\phi]$, $h[\phi_R] = k^{-1} Z_\psi^{-1} H[\phi]$. We obtain a functional system of the form

$$\dot{v} = F_1(v, v', v'', h, \bar{Z}, \dot{\bar{Z}}, \eta_\phi, \eta_\psi) \quad (10)$$

$$\dot{h} = F_2(h, h', h'', v', v'', \bar{Z}, \dot{\bar{Z}}, \eta_\phi, \eta_\psi) \quad (11)$$

If we substitute the previous relations for $\{\eta_\phi, \eta_\psi, \dot{Z}\}$, anomalous dimensions are eliminated and we remain with the system $\{\dot{v}, \dot{h}, \dot{\bar{Z}}\}$. The details of these partial differential equations, and of \dot{Z} , η_ϕ and η_ψ will be given elsewhere [22].

Now we truncate further v and h to get a finite system of ordinary differential equations. We adopt a power expansion of v , that is valid for values of ϕ_R within a scale dependent radius of convergence, to some finite order. A \mathbb{Z}_2 -symmetric phase is analyzed through the expansion $v = \lambda_0 + \lambda_2 \phi_R^2 + \lambda_4 \phi_R^4$ where $\langle \phi_R \rangle = 0$ and $\lambda_2 > 0$. Alternatively, if $\lambda_2 < 0$, we are in a symmetry breaking regime and expand $v = \theta_0 + \theta_4 (\phi_R^2 - \kappa)^2$. The crucial difference of the latter regime is that the running of the VEV $\langle \phi_R \rangle = \sqrt{\kappa}$ is determined by the requirement $v'(\sqrt{\kappa}) = 0$. In general the VEV beta function acquires gravitational correction only through the anomalous dimensions of matter fields:

$$\dot{\kappa} = -(2 + \eta_\phi)\kappa + \frac{v'''(1 - \frac{\eta_\phi}{6})\sqrt{\kappa}}{16\pi^2 v''(1 + v'')^2} - \frac{N_f h h'(1 - \frac{\eta_\psi}{5})\sqrt{\kappa}}{2\pi^2 v''(1 + h^2)^2} \Big|_{\bar{\phi}_R = \sqrt{\kappa}}$$

We further consider a simple Yukawa interaction according to $h(\bar{\phi}_R) = y \bar{\phi}_R$. In both phases, with truncations

up to fourth order in the scalar potential, the RG flow is encoded in a system of 5 ODEs. We mainly considered them in the DeDonder gauge ($\alpha = 0$, $\beta = 1$) but investigated also other choices.

Let us present the FP structure for the *symmetric phase*, which depends on the parameter N_f . In particular we find the expected fully Gaussian (GFP) as well as the Gaussian matter fixed points (GMFP), that exists for any value of N_f . The latter has $-1 < \eta_\psi < 0$ and, in the DeDonder gauge, $\eta_\phi = 0$. The ultraviolet critical surface has dimension 4 (but 3 for the fully Gaussian case for $N_f < 3$).

Moreover we find the existence of a fully non trivial FP for $N_f > 3$, with small anomalous dimensions. The dimension of the UV critical surface is always 3, independently of N_f . We present in table 1 few examples of FP and the asymptotic behavior $N_f \rightarrow \infty$. We also show the values of critical exponents c_i .

N_f	λ_0	λ_2	λ_4	y	\bar{Z}	η_ψ	η_ϕ	c_1	c_2	c_3	c_4	c_5
4	-0.0129	0.243	0.832	2.70	0.00562	-0.215	0.400	-1.02	-0.566	1.30	1.38	3.88
8	-0.0260	0.0872	0.437	1.25	0.00700	-0.102	0.225	-0.613	-0.240	1.66	1.68	3.96
20	-0.0642	0.0289	0.0543	0.467	0.0130	-0.0407	0.0958	-0.235	-0.0937	1.86	1.87	3.99
$N_f \rightarrow \infty$	$\lambda_{0,\infty} - \frac{N_f}{32\pi^2}$	$\frac{\lambda_{2,\infty}}{N_f}$	$\frac{\lambda_{4,\infty}}{N_f^2}$	$\frac{y_\infty}{N_f}$	$Z_\infty + \frac{N_f}{192\pi^2}$	$\frac{\eta_{\psi,\infty}}{N_f}$	$\frac{\eta_{\phi,\infty}}{N_f}$	$\frac{c_{1,\infty}}{N_f}$	$\frac{c_{2,\infty}}{N_f}$	$2 + \frac{c_{3,\infty}}{N_f}$	$2 + \frac{c_{4,\infty}}{N_f}$	$4 + \frac{c_{5,\infty}}{N_f}$

Tab. 1. Position of the non trivial fixed point, the critical exponents and the matter field anomalous dimensions for varying N_f and their asymptotic values. The asymptotic constants have been determined analytically [22].

The non trivial fixed point approaches the GMFP in the limit $N_f \rightarrow \infty$. We also considered scalar potential truncations with higher order polinomials. The shape of the potential at the FP changes very little and the FP exists with negligible variations for λ_0 , y and \bar{Z} for all values of N_f . Moreover the matter field anomalous dimensions appear to be small and so we expect the truncation to be reliable.

There is little sensitivity with respect to the choice of the cutoff function, as was already discussed in [19]. Adopting a pure cutoff scheme, where even the field strength of the fields is not included in \mathcal{R}_k , is leading to very similar results. Moreover we have observed a very good stability of the results on changing the gauge parameters $\alpha \geq 0$ and β . The dependence on the latter is due to the offshellness of the flat space background choice and to the truncation employed in (8).

We consider now the *SSB phase* with a non zero scalar VEV. The numerical search for the FPs has revealed five N_f -dependent families of solutions. There is a (generalized) GMFP solution ($\theta_4 = y = 0$ and $\kappa = 3/32\pi^2$) with $\eta_\phi = 0$ and typically $|\eta_\psi| \ll 1$ and UV critical dimension 4. Another solution exists for $N_f = 1, 2$ and has $y = 0$, small anomalous dimensions and is completely UV repulsive. The other three solutions, completely non trivial, are characterized by at least one large anomalous dimension and therefore are not very appealing.

IV. DISCUSSIONS

We have computed, using functional renormalization group techniques, quantum corrections in a special but non trivial interacting gravity matter system, taking into account the anomalous dimensions of the matter fields (one scalar and N_f Dirac fermions). The results we obtained corroborate the scenario of an asymptotically safe gravity. A non trivial FP in the gravitational sector is realized as GMFP, independently of the cutoff schemes used in both symmetric and broken phases.

Moreover we find that in the symmetric phase another FP exists, completely non trivial, i.e. with non constant scalar potential v and non zero Yukawa coupling associated to a three dimensional critical surface. Such a point is stable under different polynomial expansions of the local scalar potential and change of the gauge fixing parameters. We found that, when neglecting the anomalous dimensions, this FP exists for $5 < N_f \leq 12$, with couplings of the same order of magnitude, and that another non trivial one appears, with much larger values and a different N_f asymptotic dependence. The lesson we learn is that, in general, anomalous dimensions should be taken into account as this might be crucial to determine the correct FP structure. In the broken phase several non trivial FP solutions appears, but due to their large anomalous dimensions we do not consider them physi-

cally trustable.

In the previous sections we studied in detail the RG flow adopting a type-I cutoff scheme. In such a scheme, for n_s scalar, n_D Dirac fermions and n_M massless vector fields, the gravitational β function acquires a leading term proportional to $(-n_s + n_D + \frac{7}{4}n_M)$ [14]. Another often used scheme (type-II) is such that the same contribution is proportional to $(-\frac{1}{2}n_s - n_D + 2n_M)$. The contribution to the vacuum energy is the same, proportional to $(n_s - 4n_D + 2n_M)$ in both schemes [14]. Studying the RG flow of our simple model using the type-II cutoff one finds that, for any N_f , the non trivial FP disappears. However it is clear that this feature may change when introducing a more realistic matter sector. As a preliminary test we considered our simplified toy model with additional free scalar, Dirac fermion and vector fields. Having in mind to keep a Yukawa coupling for the top quark only, which belongs to a fundamental representation of $SU(3)_c$, we set $N_f = 3$ and the number of the other free fields to match the content of the standard model. Repeating the RG flow analysis, we found for both cutoff type schemes (I and II) a non trivial FP. This FP is non trivial, both in the gravitational and in the interacting scalar-fermion sector and has a Yukawa coupling with value about unity. The non trivial FP exists also when changing the number of the extra free fields around the values corresponding to the standard model content, for a wide range of parameters. We observe that typically it has a negative value λ_0 , which nevertheless may change sign in the RG

flow towards the IR. An analysis of the influence of free fields on the gravitational FP with type-I cutoff was previously presented in [17]. Details of our analysis will be given in [22].

In a scenario with an asymptotically safe gravity, there has been a proposal to predict the Higgs boson mass [23]. Negative gravitational corrections to the top Yukawa beta function were required. However, if we expand perturbatively our results and retain the leading order, we find a negative gravitational contribution to the fermion anomalous dimensions, but a positive gravity-induced term to the Yukawa beta function, $\beta_y^{grav} = a_y y G k^2$ with $a_y \sim 0.2$.

It is intriguing that in the symmetric phase we get a fully non trivial FP solution with a Yukawa coupling value around unity, which is close to the top Yukawa one at low energies. One may imagine a scenario with a modest flow of the Yukawa coupling from UV (ultra Planckian) to IR (Fermi) scales. Nevertheless, one should include a mechanism to obtain a low energy SSB phase. On the other hand a full investigation of a situation closer to the standard model might reveal the possibility that a trustable non trivial fixed point is realized even in an Higgs SSB phase. We plan to study and discuss more these issues elsewhere.

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